

# Achieving COSMOS

A Metric for Determining When to Give up and When to Reach  
for the Stars

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Achieving COSMOS :  
A Metric for Determining  
When to Give up  
and When to Reach for the Stars

Is 100 runs enough?

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Or:

Is 100 runs enough?

Or:

Have I done enough runs to make reliable inferences *using this system?*

## Koza's Measures

$M$  : population size/indexable array of ordered individual programs

$s$  : success predicate (problem-specific)

Name	Description	Formula/Estimator
$\mathcal{I}(M, i, \delta)$	Computational Effort	$\mathcal{R}(\delta) \times i \times M$
$\mathcal{R}(\delta)$	Number of independent runs needed to satisfy $s$ with probability $\delta$	$\mathcal{R}(\delta) = \left[ \frac{\log \epsilon}{\log(1-P(M,i))} \right]$
$\mathcal{P}(M, i)$	Cumulative Probability of Success	$\sum_i \mathcal{Y}(M, i)$
$\mathcal{Y}(M, i)$	Point Probability of Success	$\sum_k s(M_i(0))$

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# COSMOS





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Convergence  
of

# COSMOS



**C**onvergence  
**o**f  
**S**ample  
**M**eans for

# COSMOS



**C**onvergence  
**o**f  
**S**ample  
**M**eans for  
**O**rders  
**S**tatistics

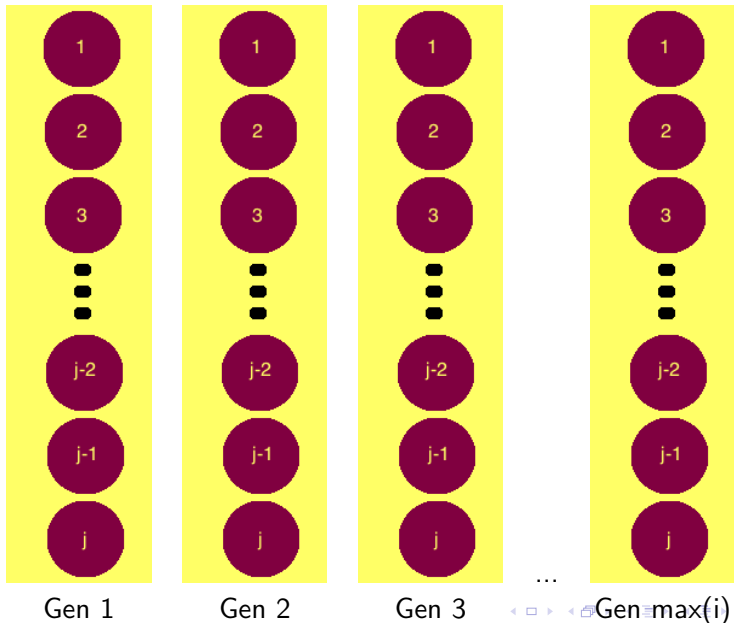
# COSMOS



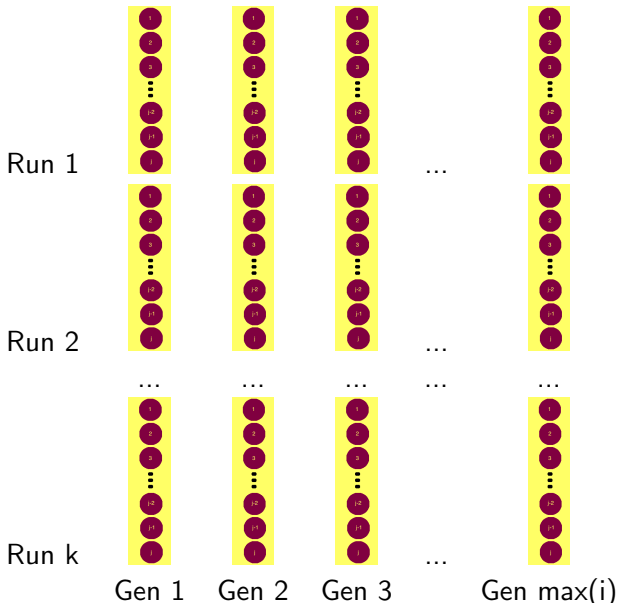
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Convergence  
of  
Sample  
Means for  
Order  
Statistics

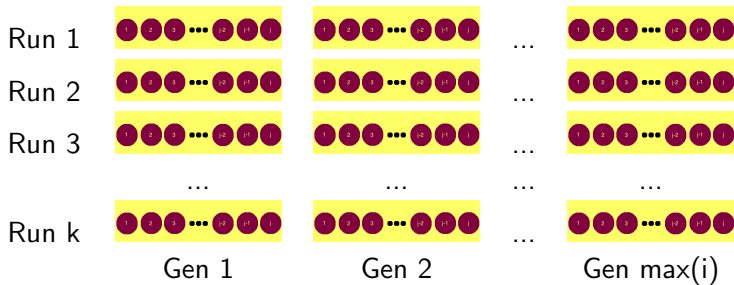
## Generational Model



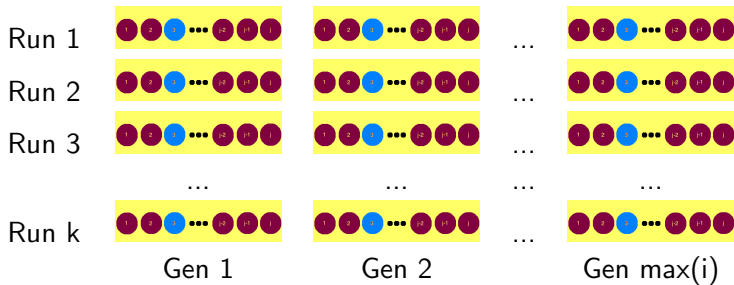
# Generational Model across $k$ runs



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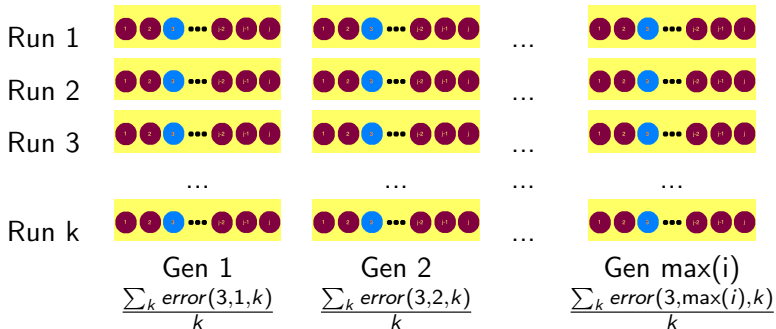


## Select the third ordered individual...





...and calculate the sample mean



What might convergence for some sample mean might look like?

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...say, the third ordinal of the second generation?

# How to define Convergence?

## Convergence for ordinal $q$ in generation $i$

Let:

$$\bar{X}_k$$

Mean of  $k$  random samples

$$\bar{X}_{k+1}$$

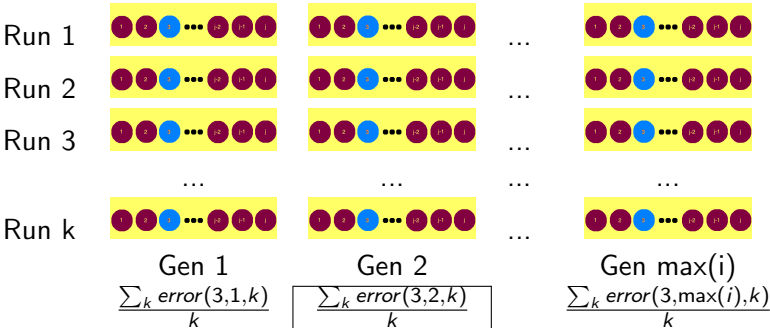
Mean of  $k + 1$  random samples

We want:

$$\bar{X}_k = (1 \pm \epsilon)\bar{X}_{k+1}$$

for some error  $\epsilon$  with high probability.

Recall...



## Convergence for ordinal 3 in generation 2

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$$\bar{X}_{k+1} = \frac{\sum_{k+1} \text{error}(3, 2, k + 1)}{k + 1}$$



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Idea: Want to detect *when* doing more runs doesn't give us any new information.

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- Means not that useful for most distributions
- Commonly used in nonparametric statistics for their robustness

## Selected Order Statistics

- min
- “lower quartile”
- “median”
- “upper quartile”
- max

# COSMOS Estimator

For each generation:

1. Create an array to hold the sample means of the ordinals.
2. For each ordinal:
  - a. Start with some minimum number of runs and compute its mean.
  - b. Add one run to the total and recompute the mean.
  - c. Take the ratio of the two sample means and enter it into the array.
3. If the ratio is within  $1 \pm \epsilon$  for all ordinals, return the number of runs.
4. Else, repeat.

Return the maximum number of recommended runs.



## Context

Repeating experiments from (Luke 2001) and (Daida 2001)

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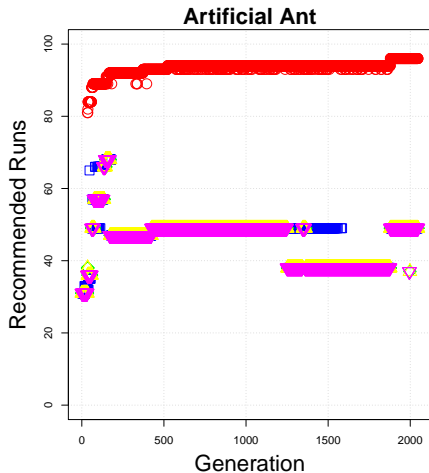
## Context

Repeating experiments from (Luke 2001) and (Daida 2001)

- All experiments used a GP System (ECJ)
- Fitness values were default scaled errors
- GP system was generational (steady-state version is simpler!)

# Artificial Ant

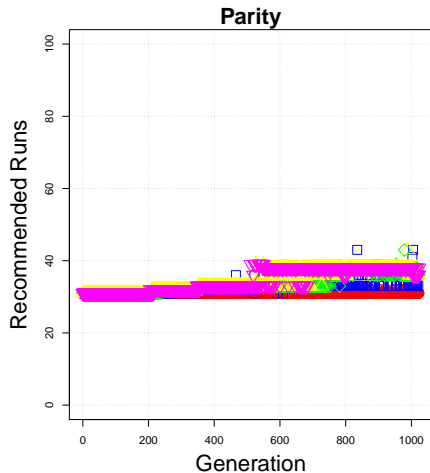
- Ordinals
- 
- Minimum
  - Lower Quartile
  - ◇ Median
  - △ Upper Quartile
  - ▽ Maximum



# Parity

## Ordinals

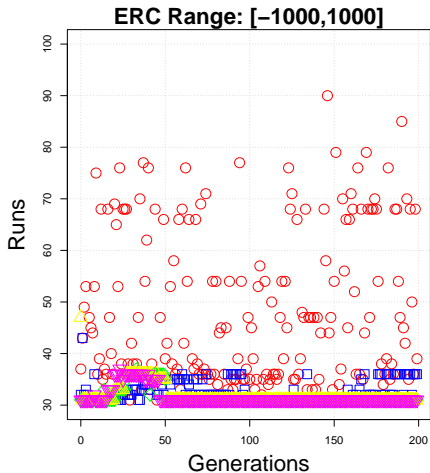
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# Binomial-3 ERC Range: [-1000,1000]

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- Lower Quartile
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# Future Work



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- Proving bounds on the estimator
- Lower bounds on runs
- Utility of non-convergence?

# Acknowledgements

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- My advisor, Lee Spector
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